

Materi : Pangkat, Persamaan Kuadrat, Fungsi kuadrat dan Bentuk Akar

1. Pangkat Eksponen

$$a^p \cdot a^q = a^{p+q} \qquad (a \cdot b)^p = a^p \cdot b^p \qquad a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

$$\frac{a^p}{a^q} = a^{p-q} = \qquad a^0 = 1$$

$$(a^p)^q = a^{p \cdot q} \qquad a^{-p} = \frac{1}{a^p}$$

2. Persamaan Kuadrat

$$ax^2 + bx + c = 0 ; a, b, c \in \mathbb{R} ; a \neq 0$$

$$x = \frac{-b \pm \sqrt{D}}{2a} ; D = b^2 - 4ac$$

Rumus jumlah dan hasil kali akar-akar persamaan kuadrat :

$$x_1 + x_2 = -\frac{b}{a} \qquad x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1 \cdot x_2$$

$$x_1 \cdot x_2 = \frac{c}{a} \qquad \frac{1}{x_1} + \frac{1}{x_2} = \frac{x_2 + x_1}{x_1 \cdot x_2}$$

$$x_1 - x_2 = \left| \frac{\sqrt{D}}{a} \right| \qquad x_1^2 \cdot x_2 + x_1 \cdot x_2^2 = x_1 \cdot x_2 (x_1 + x_2)$$

3. Menyusun persamaan kuadrat baru

$$(x - x_1)(x - x_2) = 0$$

$$x^2 - (x_1 + x_2)x + x_1 \cdot x_2 = 0$$

4. Fungsi Kuadrat

Cara melukis grafik $ax^2 + bx + c = 0$

a). Menentukan titik potong sumbu koordinat (jika ada)

* Menentukan sumbu x jika $y = 0$ $D = b^2 - 4ac$

cara menfaktorkan $(x_1, 0) ; (x_2, 0)$

* Menentukan sumbu y jika $x = 0$; $(0, c)$

b). Menentukan persamaan sumbu simetri

$$x = \frac{-b}{2a}$$

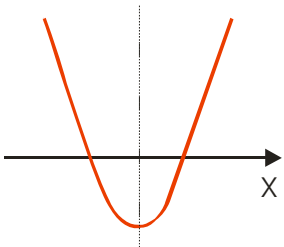
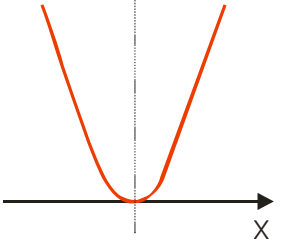
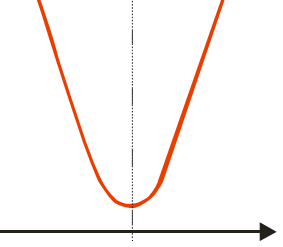
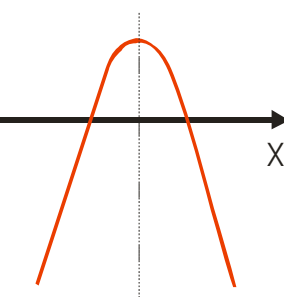
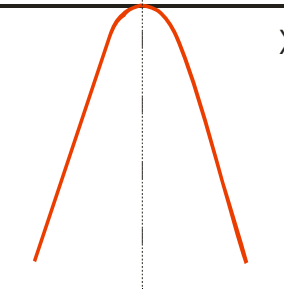
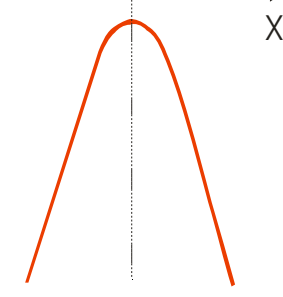
c). Menentukan nilai minimum/ maksimum

$$y_m = \frac{b^2 - 4ac}{-4a} \gg \frac{D}{-4a}$$

d). Menentukan titik puncak parabola

$$\left\{ \frac{-b}{2a}, \frac{D}{-4a} \right\}$$

5. Sketsa grafik persamaan kuadrat

	D > 0	D = 0	D < 0
a > 0	 <p>Titik Puncak Minimum (-)</p>	 <p>Titik Puncak Minimum (-)</p>	 <p>Titik Puncak Minimum (-)</p>
a < 0	 <p>Titik Puncak Maksimum (+)</p>	 <p>Titik Puncak Maksimum (+)</p>	 <p>Titik Puncak Maksimum (+)</p>

6. Bentuk Akar

$$(\sqrt{a})^2 = a = \sqrt{a^2} = a \geq 0$$

$$\sqrt{a} = a^{\frac{1}{2}}$$

$$\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$$

$$\sqrt{a} + \sqrt{b} = \sqrt{a} + \sqrt{b}$$

$$\sqrt{a} - \sqrt{b} = \sqrt{a} - \sqrt{b}$$

$$(\sqrt{a} + \sqrt{b}) + \sqrt{c} = \sqrt{a} + (\sqrt{b} + \sqrt{c})$$

$$\sqrt{(a+b)} + 2\sqrt{ab} = \sqrt{a} + \sqrt{b}$$

$$\sqrt{(a+b)} - 2\sqrt{ab} = \sqrt{a} - \sqrt{b} ; a > b$$

7. Merasionalkan Penyebut

$$\frac{a}{\sqrt{b}} = \frac{a}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{a}{b} \sqrt{b}$$

$$\frac{a}{c\sqrt{b}} = \frac{a}{c\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{a}{cb} \sqrt{b}$$

$$\frac{c}{\sqrt{a} + \sqrt{b}} = \frac{c}{\sqrt{a} + \sqrt{b}} \times \frac{\sqrt{a} - \sqrt{b}}{\sqrt{a} - \sqrt{b}} = \frac{c}{a - b} (\sqrt{a} - \sqrt{b})$$

$$\frac{c}{\sqrt{a} - \sqrt{b}} = \frac{c}{\sqrt{a} - \sqrt{b}} \times \frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} + \sqrt{b}} = \frac{c}{a + b} (\sqrt{a} + \sqrt{b})$$

$$\frac{a}{b + \sqrt{c}} = \frac{a}{b + \sqrt{c}} \times \frac{b - \sqrt{c}}{b - \sqrt{c}} = \frac{a}{b^2 - c} (b - \sqrt{c})$$

$$\frac{a}{b - \sqrt{c}} = \frac{a}{b - \sqrt{c}} \times \frac{b + \sqrt{c}}{b + \sqrt{c}} = \frac{a}{b^2 + c} (b + \sqrt{c})$$

Contoh :

$$\begin{aligned} \frac{8}{1 - \sqrt{5} + \sqrt{3}} &= \frac{8}{(1 - \sqrt{5}) + \sqrt{3}} \times \frac{(1 - \sqrt{5}) - \sqrt{3}}{(1 - \sqrt{5}) - \sqrt{3}} = \frac{8(1 - \sqrt{5} - \sqrt{3})}{(1 - \sqrt{5})^2 - 3} = \frac{8 - 8\sqrt{5} - 8\sqrt{3}}{1 - 2\sqrt{5} + 5 - 3} \\ &= \frac{8(1 - \sqrt{5} - \sqrt{3})}{3 - 2\sqrt{5}} \end{aligned}$$

8. Persamaan Kuadrat

a). Memfaktorkan

$$ax^2 + bx + c = 0$$

Contoh :

$$x^2 - 5x - 6 = 0$$

$$(x - 6)(x + 1) = 0 \gg = x \cdot x = x^2 \text{ (nilai a)} ; \quad -6 + 1 = -5 \text{ (nilai bx)} ; \quad -6 \times 1 = -6 \text{ (nilai c)}$$

$$x = 6 \text{ atau } x = -1$$

b). Melengkapkan Kuadrat Sempurna

Contoh :

Menentukan titik koordinat

$$Y = x^2 - 8x + 10$$

$$(x - 4)^2 - 16 + 10$$

$$(x - 4)^2 - 6$$

$$(y + 6) = (x - 4)^2 \rightarrow (4; -6)$$

c). Rumus abc

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

9. Menentukan KPK dan FPB

KPK (Kelipatan Persekutuan Terkecil)

FPB (Faktor Persekutuan Terbesar)

Contoh :

Tentukan KPK dan FPB dari 54 dan 60

Penyelesaian cara I :

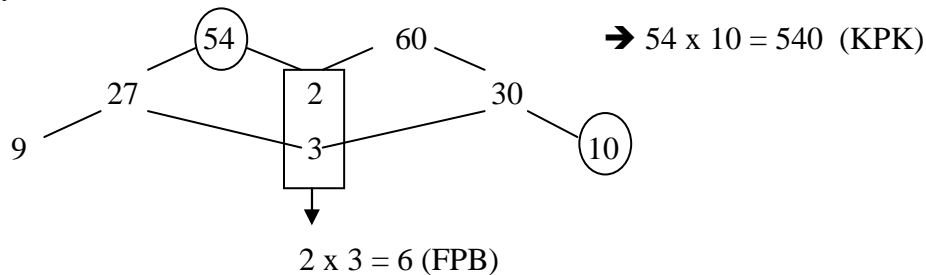
$$54 = 2 \times 3 \times 3 \times 3 = 2 \times 3^3$$

$$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$$

$$\text{KPK} = 2^2 \times 3^3 \times 5 = 540 \text{ (ambil pangkat tertinggi)}$$

$$\text{FPB} = 2 \times 3 = 6 \text{ (ambil pangkat terendah)}$$

Penyelesaian cara II :



Contoh :

Tentukan KPK dan FPB dari $4xy^2z^5$ dan $6x^2z$

$$4xy^2z^5 = 2^2 \times x \times y^2 \times z^5$$

$$6x^2z = 2 \times 3 \times x^2 \times z$$

$$\text{KPK} = 2^2 \times 3 \times x^2 \times y^2 \times z^5 = 4 \times 3 \times x^2 \times y^2 \times z^5 = 12x^2y^2z^5$$

$$\text{FPB} = 2 \times x \times z = 2xz$$